

SELECTED PROBLEMS CONCERNING THE ANALYSIS OF THIN-WALLED STRUCTURES WITH THE USE OF FINITE ELEMENT METHOD

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Abstract

The aim of the paper was presentation and comparison of numerical methods applied in thin-walled structures analysis, with special attention paid to possibility of usage the Finite Element Method (FEM), especially in nonlinear analysis. There were presented basic differences between classical approach to modelling and analyzing thin-walled structures, and these performed with FEM methods using. The biggest emphasis was placed on the analysis of semimonocoques, in which the loss in the shell's stability is possible in the range of operational load activity. According to this, many nonlinear terms like global and local buckling, contact problems, significant deformations and shifts, are present. Worth emphasizing is the fact, that tension field, is something that was described long time ago. First papers about this phenomenon were published in the last century. Although it is a common effect that takes place in semimonocoques, there are not many publications that analyze and examine this phenomenon. In this paper, an analysis of two-sided, thin-walled spar, which has undergone the flexion by the shearing force in the plane of the panel, was presented. The spar was designed as a classic semimonocoque, in which load transmission function is separated. It was assumed that, for the sake of small thickness, shell elements transmit mainly tangential loads, normal loads, when normal loads are transmitted by framework elements. Riveted joints are used to join elements of framework and shell. There are presented and compared results of analysis for models with various complexities. Firstly, results of analysis of classical model of semimonocoques were presented. It means that framework elements were modelled as rod elements transmitted only normal loads, but shell elements were modelled as disc semimonocoque elements, in this case transmitted only tangential loads. Area of section of rod elements was adequately increased, in order to consideration mating width, coming from partial transmission of normal loads by the shell elements. Results obtained for intermediate models were also presented, as well as for complex shell model, which allows on advanced nonlinear analysis of tension field, which consider contact between framework and shell elements. Applications of formulated models to thin-walled structure, especially aircraft ones and further possibilities of presented method of analysis were discussed.

Keywords: *diagonal tension, wing structures, thin-walled structures, aircraft, FEM modelling*

1. Introduction

Presented paper is the first part of the work concerning the analysis of the influence of tension field on riveted joints, which are used in thin-walled structures. The aim of this work is to present and compare the analytical methods used during analyzing thin-walled constructions, with special attention paid to possibility of usage the Finite Element Method (FEM), especially in nonlinear analysis, illustrated with the example of tension field effect. The biggest emphasis is placed on the

analysis of semimonocoques, in which the loss in the shell's stability is possible in the range of operational load activity. According to this, many nonlinear terms like global and local buckling, contact problems, significant deformations and shifts, are present.

In this paper, an analysis of two-sided, thin-walled spar, which has undergone the flexion by the shearing force in the plane of the panel, is presented (Fig. 1). The spar mentioned above can be either a wing spar, or a component of a transport aircraft fuselage (Fig. 2). Shearing load, favours the formation of diagonal tension (tension field)¹. The spar was designed as a classic semimonocoque, in which the exceeding of the critical force is allowed, what causes buckling of the shell.

In relation to the above mentioned the results of analyses, of the structures with various complexities, are presented and compared in this report.

At first measurement and analytical results of the classic semimonocoque are presented. It means that in mentioned structure, components of the framework were modelled as frames able to transmit normal stresses, and components of the shell were modelled as plate structures, that transmit only shearing stresses. The section area of frames was enlarged because it was necessary to take into consideration the mating surface, which results from the fact that normal stress is partly transmitted by the shell (Marquerre, Stowell, Lahede, Wagner and many others authors described it in their reports) [10].

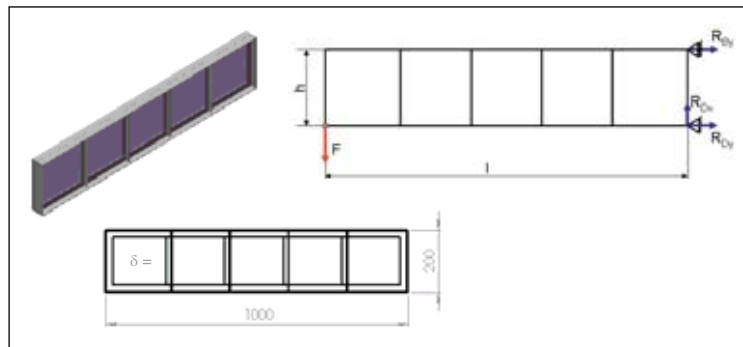


Fig. 1. Analytical model and scheme

Worth emphasizing is the fact, that tension field, is something that was described long time ago. First papers about this phenomenon were published in the last century [16, 20]. Although it is a common effect that takes place in semimonocoques, there are not many publications that analyze and examine this phenomenon. This could be confirmed by the document, published by NASA² in 2002, which consists of instructions for designing aircraft structures [17].



¹ Tension field (diagonal field) - particular case of stability loss of shells of thin-walled structures as a result of shearing force activity [4]-more in section 3.

² NASA - National Aeronautics and Space Administration.

Fig. 2. a) Transport aircraft PZL M-28 Skytruck, flying in Venezuela aircraft fleet¹, b) framework of the fuselage²

Consequently results of findings about transitional models were presented. Finally, analytical results of complex shell-plate model, which is the base for advanced nonlinear analysis of the tension field, including the contact between the components of the framework and the shell.

2. Thin-walled structures

Thin-walled structures are very common in the surrounding world. These constructions are described as mechanical structures, and what characterizes them, is the fact that one of the dimensions of the main structural components is much smaller than other dimensions [4, 12, 16]. Typical examples are aircraft constructions (Fig. 2). The evolution of the aircraft constructing, in the beginning of the 20th century, caused the growth of interest, and was the driving force for the evolution of thin-walled structures analyzing. First reports were published over the years 1936-1939 by H. Wagner and H. Simonn [20, 21], and H. Köller [8, 9]. Despite the fact that the reports mentioned above, were about specific models and specific types of loads, were still a ground for further analyzing of thin-walled structures.

In years 1944-1949, other reports appeared, worth mentioning are reports published by P. Cical [5, 6], who introduced a semimonocoque, which is now considered as a classic approach to thin-walled structures modelling. Since that time till 1950`s, several reports concerning the discussed matter have appeared.

In 1956 P. Kuhn published the monograph [16], which can be treated as a summary of up to now achievements in the field of thin-walled constructions analysis and investigations. The stability and the static of the construction are discussed in details in this report. Major part of this monograph is devoted to the analysis of the tension field effect. This publication contains also many findings and thorough, theoretical analysis of this phenomenon.

Other reports and publications on this field, both Polish and non-Polish, were based on the achievements of Wagner, Simonn [20], Ebner, Köller [8, 9], Cical [5-7] and Kuhn [15, 16]. Among Polish publications, based on the achievements of the authors mentioned, the most significant one is the work published by Professor Zbigniew Brzoska [4].

True thin-walled structures are usually complex constructions, which are exposed to complex loads, and this is the reason why in many cases it is impossible to obtain an exact result of the analysis. It is common to use simplified models to analyze such complex constructions. Despite certain theoretical achievements, in the field of static analysis of thin-walled structures, due to multitude of equations it was still difficult and sometimes impossible to obtain an exact solution.

A significant progress took place in the 60`s and 70`s and was caused by the development of the matrix methods to analysis of various structures. This progress enabled the rise of Finite Element Method.

However these analytic methods were fully used after the development of computer technologies in 70`s and 80`s [11, 14, 19]. Nowadays Finite Element Method (FEM) is the main technique used in analyzing thin-walled structures and it seems to be more and more common technique used in analyzing all kinds of constructions [1].

Discussed constructions appear in numbers of publications and various branches of technique, and therefore the problem of meaning appears. It applies to misuse of the proper names of thin-walled constructions, and the names of the methods of analyzing [4, 12, 16, 18, 22]. Classic thin-walled structures can be divided into:

- semimonocoques – in these structures the components of the shell are very thin and the loss in the stability of the shell is possible in the elastic range of operational loads activity. Characteristic features of these structures are: small thickness of the shell relatively dense

¹ Photo from <http://www.rescate.com/GN97121.html>

² Picture from <http://www.cad.pzlmilec.pl/>

structure of the framework, the local loss in the stability of the shell is possible in the elastic range. Due to small thickness of the shell (skin), its elements are under membrane stresses rather than bending stresses, bending moments, and local bendings are taken over by the components of fuselage (bending stresses in the components of the shell are not significant, and usually do not exceed 5%),

- monocoques – in these kinds of structures the loss in the stability of the shell, is not acceptable in the elastic range of operational loads activity. Characteristic features of these structures are: big thickness of the shell layer (as far as thin-walled structures are concerned), relatively thin structure of the framework.

Whereas analytical models of thin-walled structures can be divided into:

- semimonocoque model – a model in which bending stresses are not taken into consideration, and clear division of stress transmitting (load transmitting) is introduced: Components of the framework transmit only normal forces (section fields of the elements of framework are enlarged with the number that reflects the participation of the shell in transmitting normal loads). Components of the shell transmit only contact forces. Joints of the framework are treated as ideal ones,
- monocoque model – a model in which bending stresses are taken into consideration and it is assumed that the shell is under the membrane stress or flat stress.

Shell-plate model-a model, in which the full analysis of bending stress is made, both for components of the framework and the shell.

3. Tension field

Among thin-walled constructions, the most significant are semimonocoques, the typical examples of such constructions, are aircraft structures. As it has been already mentioned, in these constructions the loss in the stability of the shell is possible in the elastic range of operational loads activity. For instance in Skytruck PZL M-28¹ (Fig. 2) the buckling of the fuselage can occur just after the exceedance of 40% of maximum operational load.

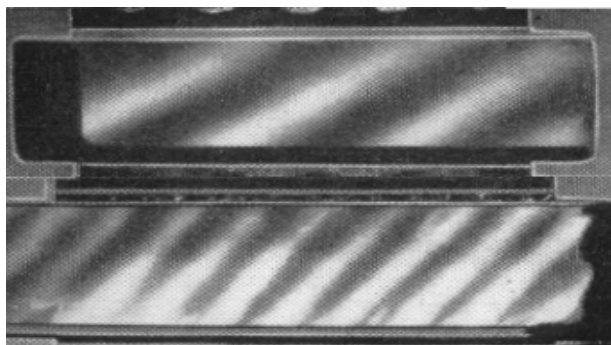


Fig. 3. Tension field. Photograph of tension field from experiment made by H. Wagner [10]

Usually buckling is the result of normal stresses activity (bending or axial compression of the shell) and shearing is a related phenomenon that causes the reduction of critical force. In case of diagonal tension (tension field), loss in the stability of the shell is the result of shearing forces activity. This is common for fuselages exposed to the torsion stress, and wing spars in which shells are exposed mainly to shearing stress.

As it was mentioned, the diagonal tension is a special case of the impairment of stability of the shell in thin-walled constructions. This impairment is caused by inertial forces. In Polish this phenomenon is called “pole ciągnień”. In 1928 H. Wagner [20] conducted an experiment; he

¹ PZL M-28 Skytruck – airplane produced by Polish aircraft factory PZL Mielec

subjected thin shell, reinforced with diagonal frames, to the load, what made the surface folded (Fig. 3). He proved that the buckling of the shell does not destroy the construction, until the lateral reinforcements (transverse frames) are exposed to shearing stresses (until the stability of reinforcements is lost). This experiment had a big impact on further designing of thin-walled structures.

Still there are not many publications that deal with the impact of the tension field on the effort of semimonocoques. The tension field effect may be the reason for many local phenomena that cause accumulation of stresses in various joints. This mainly applies to riveted joints and bolted joints that are still commonly used in aircraft structures. In relation to the above mentioned, many researches and calculations have been made to describe the impact of tension field effect on the stresses and deformations that appear in the place of contact of the framework and the shell.

4. Analysis of thin-walled two-sided wing spar

In order to evaluate impact of tension field on effort of the thin-walled two-sided wing spar, it was conducted some analysis, which simultaneously served to further evaluation of possibilities to use various analytical models.

It was assumed that analyzed wing spar (Fig. 1) was made of PA7 alloy (Tab. 1). Outer elements of framework (belts) were made of angle plate 25x25x3 located symmetrically on both sides of the shell. Drag struts were made of angle plate 18x18x2 and placed only on one side of the shell, causing whole structure of the wing spar asymmetrical. It should be consider during defining boundary conditions. In order to avoid torsion of the wing spar, "external" nodes moving in perpendicular direction to the shell surface was blocked. This situation corresponds to true working conditions of the framework, both in the case of working external shells and spars.

Tab. 1. Properties of PA7 alloy (AlCu4Mg2)

Parameter	Unit	Value
Tensile strength R_m	[MPa]	470
Field point R_e	[MPa]	370

All analyses have been performed with the aid of Abaqus 6.5.4 programme.

In order to execute stated task, there have been made six models and six sets of analysis respectively.

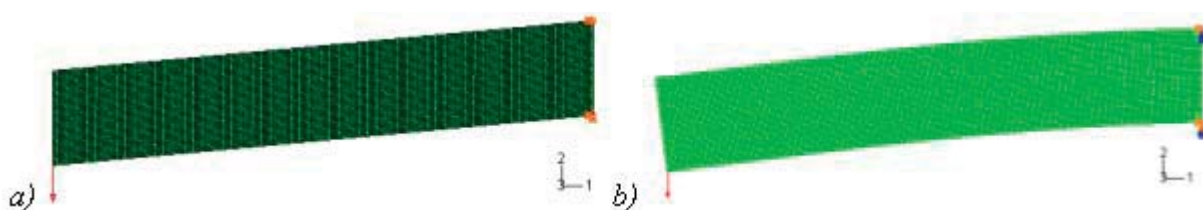
Analysis performed in the linear range allowed for models verification and load selection.

Preliminary load of wing spar was selected like that reduced stresses¹ for linear analysis were not exceeded 60% of **yield point**. (1).

$$\sigma_{dop} \leq 0.6 \cdot R_e \approx 222 \text{ MPa} , \quad (1)$$

Moreover, results of these analyses were used as a reference point for nonlinear analysis.

Static linear analyses were performed only for two models: semimonocoque and monocoque.



¹ Reduced stresses – if it is not precisely specified, what reduced stresses are consider, it should be take into consideration reduced stresses calculated according with Huber-Mises criterion.

Fig. 4. Linear analysis, deformation as a result of applied load: a) semimonocoque model, b) monocoque model

Analysis of semimonocoque model is presented in Fig. 4a. Forming semimonocoque models one should remember to consider mating width in sections of rod elements. Results obtained for analysis of semimonocoque model were applied as basic comparing results for successive, especially nonlinear analysis.

Analysis of monocoque model is presented in Fig 4b. Obtained results are concurrent with results obtained for semimonocoque model. Differences have been found locally and are related with restrain impact, and consideration of beam elements bending. Bending elements were generated on common nodes with the shell elements.

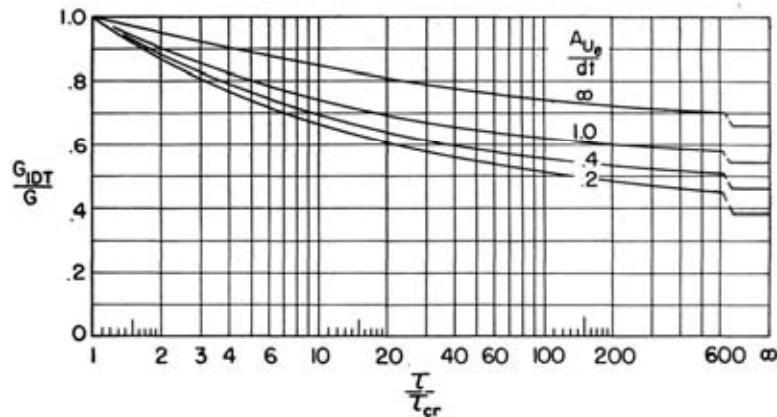


Fig. 5. Diagram presented relationship of substitute stiffness from the ratio of true shearing stresses τ to critical shearing stresses τ_{cr} [16]

Classical methods of analysis allow to simplified analysis of tension field using semimonocoque model, consider a change of shear rigidity, when overflow critical force. It could be obtained for calculations applying iteration method by changing Kirchoff modulus [4, 16] GIDT (Fig. 5) or changing shell's thickness in each step. An idea of equivalent elements applying in FEM analysis was described in [13]. Nevertheless, presented solution has been not implemented in commercial FEM systems. According with this, presented method is relatively time-consuming. Moreover, it allows only for stresses determination, whereas, for the sake of semimonocoque model specific, does not allow for evaluation of displacement in perpendicular direction to the shell. In relation with these remarks, it should be necessary to analyze tension fields in nonlinear range.

For the other models analysis included critical forces determination (linear task), and static nonlinear analysis aimed at effort of the structure investigation for the load increasing from 0 to the maximum value, especially when critical force was exceeded.

Analysis of beam-shell model is presented in Fig. 6a. In this model, as in the previous case, elements of framework were generated as beam elements with common nodes with the shell elements. Important stage of each static nonlinear analysis is selection of integration step. Obtained results confirmed expected increasing of stresses' and strains' values, when critical force was exceeded. The character of deformation responds to true deformations of similar structures.

Analysis of shell model "non-contact is presented in Fig 6b. Solid shell model was been performed, in which elements of framework were generate as a shell elements, proper distance coming from thickness of the shell and steel sections of framework was reminded. Framework and shell were joined permanently (like they be glued) using TIE¹ function. As in the case of former

¹ TIE – function accessible in Abaqus system, allowing on joining of two various sets of nodes, placed on adherent plates, with possibility of keeping defined distance.

analysis, there was important a proper selection of integration step, and results confirmed expectation concerning increasing of stress and deformation, upon critical loads exceedance.

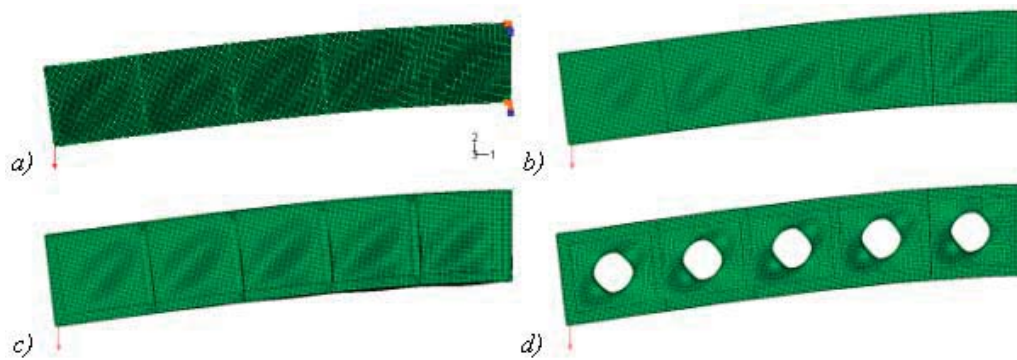


Fig. 6. Nonlinear analysis, deformation as a result of applied load, beam-shell model: a) shell model “non-contact”, b) shell model “partially contact”, c) shell model with stiffened holes

Analysis of shell model (partially contact) is presented in Fig 6c. Model with approximate mapping of riveted joint. Elements of framework were joined permanently (also using TIE function) to the shell, only on the half of width of angle bar’s shelves. A contact was model at the remainder part. It allowed for assessment of impact of free part of shelves on plate.

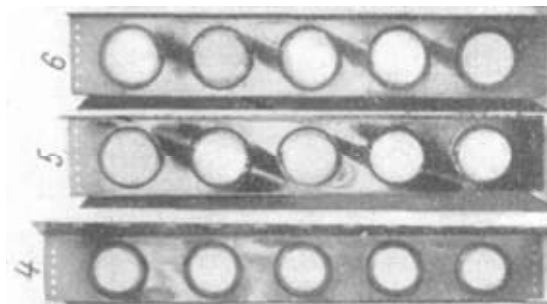


Fig. 7. Exemplary results of analysis conducted by SAAB, published by Hertel in 1960 year [10]

Analysis of shell model with stiffened holes is presented in Fig. 6d. There was performed model applying holes with appropriately shaped edges. The goal of such operation was to increase stiffness of the shell and increasing of critical force, as a consequence. Results of analysis of described model were consistent with experimental data [10].

In the Fig. 7, there are presented three forms of tension field at webs of spars with holes. Results of simulation acquired by authors correspond to the form marked on the figure as 5.

Setting of critical forces acquired for models analysed nonlinearly are presented in Tab. 2.

Tab. 2. Setting of critical forces

Model	Unit	Value
Beam-shell	[N]	3534.4
Shell „non-contact”	[N]	5748.9
Shell „partially contact”	[N]	4889.1
Shell with stiffened holes	[N]	7753.4

5. Summary of analyses

Analyzing results of load growing as a function of deflection, it could be noticed that, practically, in this case, does not to be occurred change of inclination angle of a curve, when critical force is exceeded (Fig. 8). It testifies that if relation of shell stiffness to framework stiffness

is suitable, folding of the shell does not exert an influence on global stiffness of the wing spar. Observation above, confirms a possibility of semimonocoque structures work at elastic range, after overflowing critical force.

Analysis of stresses and deformations of a section located near one of middle strut is presented in Fig. 9. As it is seen, even at long distance from place of fixing, where influence of it is negligible, there was noticed considerably impact of tension field on the structure effort. Increment of stress, both reduced and tangential, in dependence of analyzed model vary from 30% to 60% in comparison to linear analysis.

Shell deformation occurring in perpendicular direction to the shell surface, in dependence of the model, are from 0.065 mm to 0.29 mm. These data testify good index of material utilization by increasing of mean operational stresses in air structures. Air structures are first of all dimensioned by buckling.

Minimum global stiffness was determined for model with stiffened holes.

6. Conclusions

Conducted analyses allowed to obtain very interesting results and conclusions.

Discussed problems belong to strongly nonlinear group of matter and application FEM methods to solve those gives good results.

Acquired results confirmed semimonocoque structure idea, where admitted is (or even assumed) work above critical load for the shell. It allows receiving, in this way, decreasing of structure mass, and exceeding of critical load does not result in significant decreasing of global stiffness.

Tension field have essential influence on structure effort in the regions joining framework and shell, it could add to arising to stress concentration in joints. Together with substantial deformations in mentioned regions and with consideration of repeatability of this phenomenon, it could have influence on joints life.

Application of stiffened holes caused increasing of critical force, but not allowed on total eliminating of buckling, because permissible load value was $F_{per.}=10$ kN. Use of such solution had significant impact on critical force growth; however not always eliminate presence of overlap buckling. Side-effect was increasing of common deformability of the wing spar. Moreover, such solution could not be applied in case of external shells.

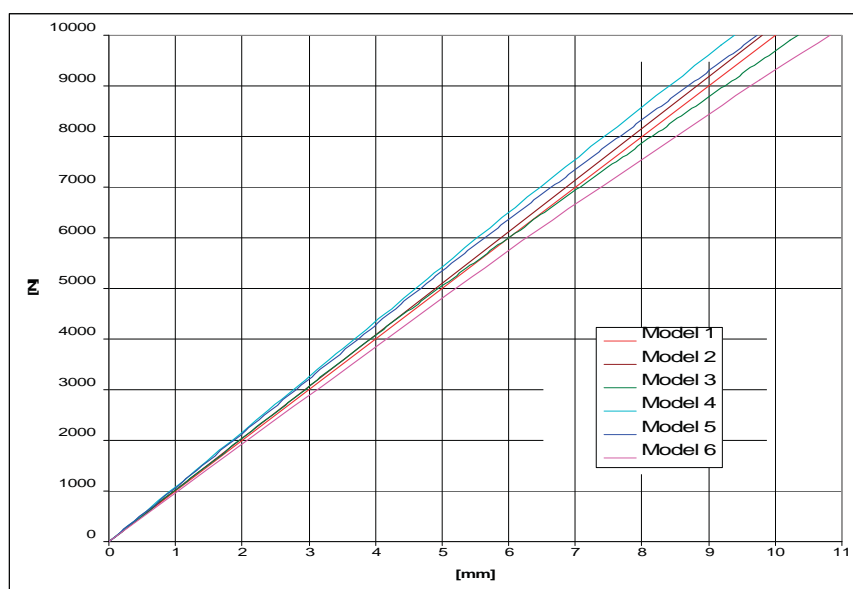


Fig. 8. Load as function of deflection, acquired for successive models

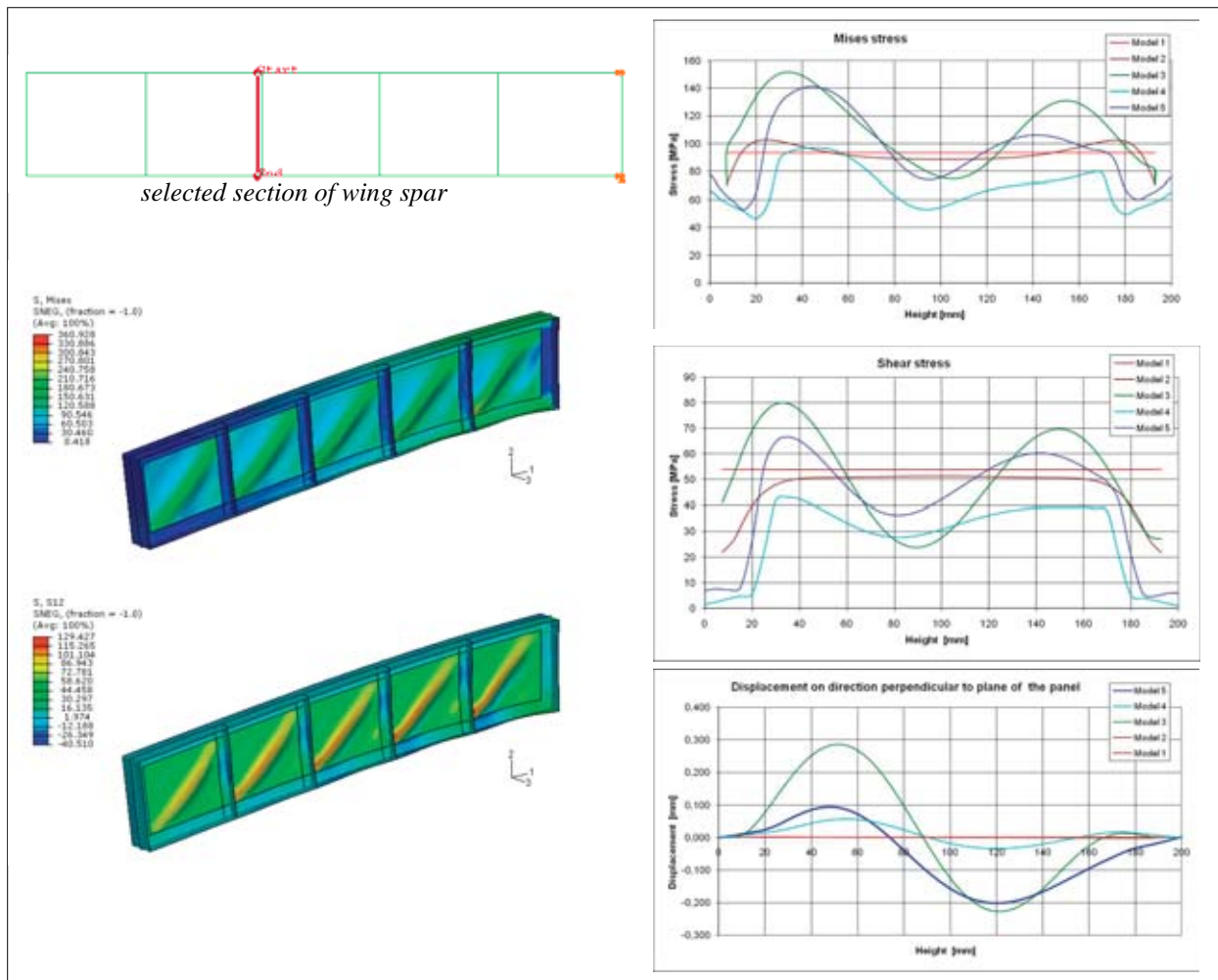


Fig 9. Results of analysis obtained for selected section of wing spar

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